Derivada del logaritmo

martes, 28 de abril de 2020 10:30

Repasar

$$f(x) = 5 \cdot 2^{3x^3+8x^2+9x}$$

$$f'(x) = 5 \cdot \ln 2 \cdot 2^{3x^3+8x^2+9x} \cdot (3x^3+8x^2+9x)' =$$

$$f'(x) = 5 \cdot \ln 2 \cdot 2^{3x^3+8x^2+9x} \cdot (9x^2+16x+9)$$

DERIVADA de la función logarítmica

$f(x) = \ln x \qquad f'(x) = \frac{1}{x}$ (Ya no hay logaritmo)

$f(x) = \log x = \log_{10} x \qquad f'(x) = \frac{1}{x \ln 10}$

$f(x) = \log_2 x \qquad f'(x) = \frac{1}{x \cdot \ln 2}$

Supongamos que no conocemos la función logaritmo

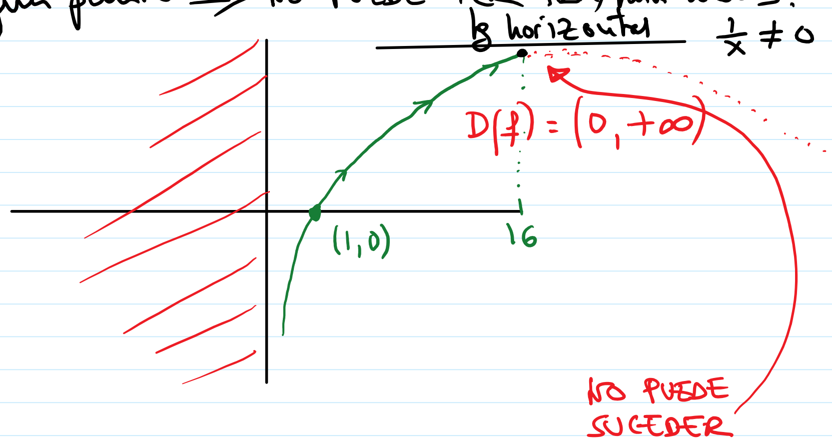
¿Tiene algún MAX/min RELATIVO el log?

$f(x) = \ln x \qquad f'(x) = \frac{1}{x}$

$f'(x) = 0 \qquad \frac{1}{x} = 0 \Rightarrow 1 = 0$ FALSO

Es decir $f(x) = \ln x$ NO TIENE TANGENTE HORIZONTAL

en ningún punto $\Rightarrow$ NO PUEDE TENER MAX/min locales.



$f(x) = \ln x \qquad f'(x) = \frac{1}{x}$

$f(x) = \ln(5x) \qquad f'(x) = \frac{5}{5x} = \frac{1}{x}$

$f(x) = \ln(2000x) \qquad f'(x) = \frac{\cancel{2000}}{\cancel{2000}x} = \frac{1}{x}$

$f(x) = \ln(2000x) \qquad f'(x) = \dfrac{\cancel{2000}}{\cancel{2000}x} = \dfrac{1}{x}$

$f(x) = \ln(7x) = \underbrace{\ln 7}_{(cte)} + \ln x = 1'945 + \ln x$

$\downarrow D$

$0 + \dfrac{1}{x} = \dfrac{1}{x}$

$f(x) = \ln(x+2) \qquad f'(x) = \dfrac{(x+2)'}{x+2} = \dfrac{1}{x+2}$

Argumento del ln

$f(x) = \ln(x^2) \qquad f'(x) = \dfrac{(x^2)'}{x^2} = \dfrac{2\cancel{x}}{x^{\cancel{2}}} = \boxed{\dfrac{2}{x}}$

$f(x) = \ln(6x^2+5x-3) \qquad f'(x) = \dfrac{(6x^2+5x-3)'}{6x^2+5x-3} =$

$f'(x) = \dfrac{12x+5}{6x^2+5x-3}$

REGLA

$f(x) = \ln(\blacksquare) \qquad f'(x) = \dfrac{\blacksquare'}{\blacksquare}$

$f(x) = \log(\blacksquare) = \log_{10}(\blacksquare) \qquad f'(x) = \dfrac{\blacksquare'}{\blacksquare \ln 10}$

$f(x) = \ln(e^x+x) \qquad f'(x) = \dfrac{(e^x+x)'}{e^x+x} = \boxed{\dfrac{e^x+1}{e^x+x}}$

OBSERVACIÓN

$f(x) = \ln(x^2) \qquad f'(x) = \dfrac{2x}{x^2} = \boxed{\dfrac{2}{x}}$

$g(x) = 2 \cdot \ln x \qquad ¿f = g?$ NO.

$\hookrightarrow g'(x) = 2 \cdot \dfrac{1}{x} = \boxed{\dfrac{2}{x}}$

MISMA DERIVADA

$f(x) = \ln(x^2) \qquad D(f) = (-\infty, 0) \cup (0, +\infty) = \mathbb{R} - \{0\}$

$f(-5) = \ln(25) = 3'21$

$f(-10) = \ln(100) = 4'6$

$f(0) = \ln(0^2) = \ln(0) =$ ERROR

$g(x) = 2 \cdot \ln x \qquad D(g) = (0, +\infty) = \mathbb{R}^+$

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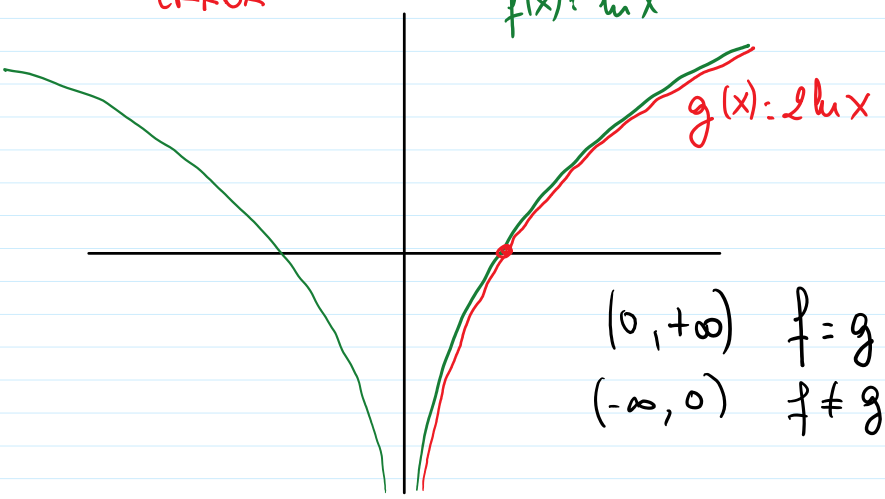
$g(-5) = 2 \cdot \ln(-5)$ no existe

ERROR



$(0, +\infty) \quad f = g \Rightarrow f' = g'$

$(-\infty, 0) \quad f \neq g$

¡MUCHO CUIDADO CON CAMBIAR/SIMPLIFICAR LAS EXPRESIONES DE UNA FUNCIÓN!

$f(x) = \ln(x^4) \neq g(x) = 4\ln x$

$f(x) = \dfrac{x^2 - 2x + 1}{x - 1} = \dfrac{(x-1)^{\cancel{2}}}{\cancel{x-1}} = x - 1$

¡MAL!

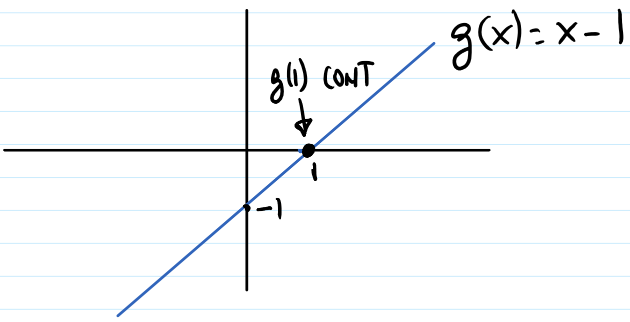
$f(x) = \dfrac{x^2 - 2x + 1}{x - 1} \qquad D(f) = \mathbb{R} - \{1\} = (-\infty, 1) \cup (1, +\infty)$

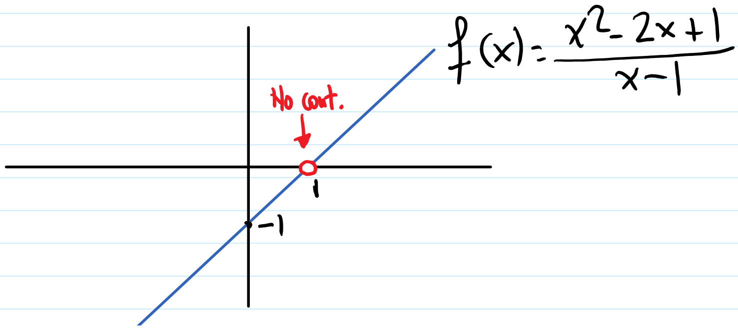
Simplificar

$g(x) = x - 1 \qquad D(g) = \mathbb{R}$

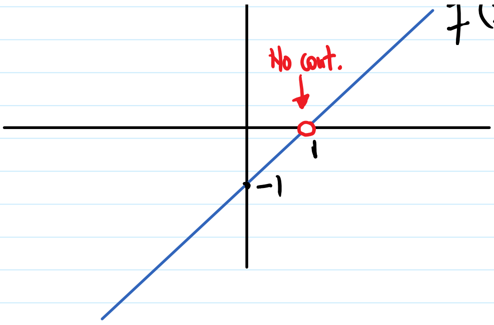
(f y g no son iguales)

f y g COINCIDEN EN TODOS LOS PUNTOS EXCEPTO $x = 1$





$f(x)= \dfrac{\ldots}{x-1}$



d) $s(x)=\dfrac{x^3+3x^2-5x+3}{x} = \dfrac{x^3}{x}+\dfrac{3x^{\cancel{2}}}{\cancel{x}}-\dfrac{5\cancel{x}}{\cancel{x}}+\dfrac{3}{x}=$

$s(x)= x^2+3x-5+\dfrac{3}{x}$

$\left(\dfrac{3}{x}\right)' = 3x^{-1}$

$\xrightarrow{d} (-1)3x^{-2} = \dfrac{-3}{x^2}$

$2x+3-0+\left(\dfrac{3}{x}\right)'$

$$\boxed{s'(x)=2x+3-\frac{3}{x^2}}$$

$$s'(x)=\frac{(3x^2+6x-5)x-(x^3+3x^2-5x+3)\cdot 1}{x^2} = (\text{cont.})$$

$f(x)=\dfrac{\sqrt{x}}{x+\sqrt[3]{x}}$ ¡horror!

→ Álgebra