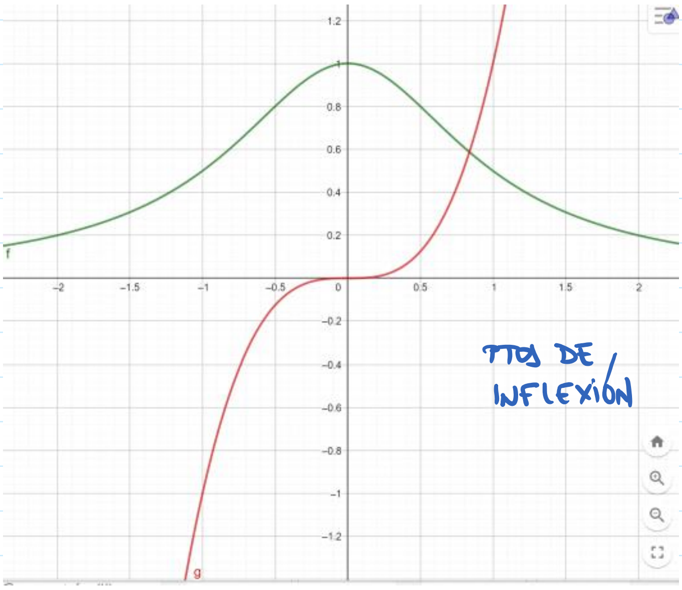
Derivadas trigonométricas

miércoles, 27 de mayo de 2020 11:28



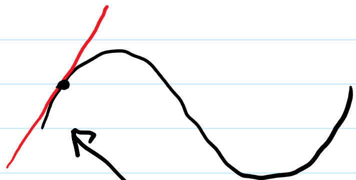
F. TRIGONOMÉTRICAS

$$\text{sen}\, x \xrightarrow{d} \cos x$$

$$\cos x \xrightarrow{d} -\text{sen}\, x$$

$$\text{tg}\, x \longrightarrow 1+\text{tg}^2 x = \frac{1}{\cos^2 x} = \left(\frac{1}{\cos x}\right)^2 = \sec^2 x$$

Recta tg en $x = \frac{\pi}{4}$ $\quad f(x) = \text{sen}\, x \quad f'(x) = \cos x$



$$y - f\left(\frac{\pi}{4}\right) = f'\left(\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right)$$

$$y - \text{sen}\,\frac{\pi}{4} = \cos\frac{\pi}{4}\left(x - \frac{\pi}{4}\right)$$

$$y - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$$

$$f(x) = \frac{\cos x}{\text{sen}\, x} \longrightarrow f'(x) = \frac{(\cos x)'\,\text{sen}\, x - \cos x \cdot (\text{sen}\, x)'}{\text{sen}^2 x} =$$

$$\shortparallel$$

$$f(x) = \text{cotg}\, x$$

$$= \frac{-\text{sen}\, x \cdot \text{sen}\, x - \cos x \cos x}{\text{sen}^2 x} =$$

$\text{sen}^2 + \cos^2 = 1$

$$= \frac{-\text{sen}^2 x - \cos^2 x}{\text{sen}^2 x} = \frac{-1}{\text{sen}^2 x}$$

$$\frac{\quad}{\operatorname{sen}^2 x} = \frac{\quad}{\operatorname{sen}^2 x}$$

$$\frac{1}{\operatorname{sen} x} = \operatorname{cosec} x \qquad \| \qquad -\operatorname{cosec}^2 x$$

TABLA

$$\operatorname{cotg} x \xrightarrow{d} -\operatorname{cosec}^2 x$$

$$f(x) = \underbrace{x^2 \cdot \operatorname{sen} x}_{\text{PRODUCTO}} \; [-] \; \underbrace{x \cos x}_{\text{PRODUCTO}} \rightarrow \text{P. tg } x = \frac{\pi}{2}$$

$$\underbrace{(x^2)'}_{\text{DERIVAR}} \operatorname{sen} x + x^2 (\operatorname{sen} x)' - \underbrace{(x)'}_{\text{DERIVAR}} \cos x + x \cdot (\cos x)'$$

$$2x \operatorname{sen} x + x^2 \cos x - 1 \cdot \cos x + x(-\operatorname{sen} x)$$

$$2x \operatorname{sen} x + x^2 \cos x - \cos x + x \operatorname{sen} x \rightarrow$$

$$3x \operatorname{sen} x + \cos x (x^2 - 1) \quad ✓$$

en $\frac{\pi}{2}$

$$2 \cdot \frac{\pi}{2} \operatorname{sen} \frac{\pi}{2} + x^2 \cancel{\cos \frac{\pi}{2}} - \cancel{\cos \frac{\pi}{2}} + \frac{\pi}{2} \operatorname{sen} \frac{\pi}{2}$$

(Revisar)

$$\pi - \frac{\pi}{2} = \frac{\pi}{2} \qquad f'\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$$

REGLA DE CADENA

$$f(x) = e^x \qquad f'(x) = e^x \qquad f'(x) = (x)' e^x = 1 \cdot e^x$$

$$f(x) = e^{x^2} \qquad f'(x) = (x^2)' e^{x^2} = \underline{\underline{2x e^{x^2}}}$$

$$f(x) = e^{\frac{1}{x}} \qquad f'(x) = \left(\frac{1}{x}\right)' e^{1/x} = \frac{-1}{x^2} e^{1/x} = \frac{-e^{1/x}}{x^2}$$

$f(x) = e^{1/x}$ $\qquad$ $f'(x) = \left(\frac{1}{x}\right)' e^{1/x} = \frac{-1}{x^2} e^{1/x} = \frac{-e^{1/x}}{x^2}$

$g(x) = 1/x$ $\qquad$ $\hookrightarrow \frac{-1}{x^2}$

$f(x) = e^{\sqrt{x}}$ $\qquad$ $f'(x) = (\sqrt{x})' e^{\sqrt{x}} = \frac{1}{2\sqrt{x}} e^{\sqrt{x}} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

$f(x) = \operatorname{sen} x$ $\qquad$ $f'(x) = \cos x$

$f(x) = \operatorname{sen}\left(\frac{1}{x}\right)$ $\qquad$ $f'(x) = \left(\cos \frac{1}{x}\right)\left(\frac{1}{x}\right)' = \frac{-1}{x^2} \cos \frac{1}{x}$

$g(x)$

$f(x) = \sqrt{x}$ $\qquad$ $f'(x) = \frac{1}{2\sqrt{x}}$

$f(x) = \sqrt{\frac{1}{x}}$ $\qquad$ $f'(x) = \frac{1}{2\sqrt{\frac{1}{x}}} \left(\frac{1}{x}\right)' = \frac{1}{2\sqrt{\frac{1}{x}}} \left(-\frac{1}{x^2}\right) = \frac{-1}{2x^2\sqrt{\frac{1}{x}}}$

(*) $f(x) = \sqrt{\operatorname{sen} x \cdot \cos x}$

RAIZ

$f'(x) = \frac{1}{2\sqrt{\operatorname{sen} x \cdot \cos x}} \underbrace{(\operatorname{sen} x \cdot \cos x)'}_{\text{PRODUCTO}} =$

$= \frac{1}{2\sqrt{\operatorname{sen} x \cos x}} \left(\cos x \cos x + \operatorname{sen} x (-\operatorname{sen} x)\right) =$

$= \frac{1}{2\sqrt{\operatorname{sen} x \cos x}} \left(\cos^2 x - \operatorname{sen}^2 x\right)$

$f(x) = \cos x$ $\qquad$ $f'(x) = -\operatorname{sen} x$

$f(x) = \cos(x^2 + 3x)$ $\qquad$ $f'(x) = -\operatorname{sen}(x^2+3x)\,(x^2+3x)' =$

$g(x)$ $\qquad$ $= -\operatorname{sen}(x^2+3x)\cdot(2x+3) =$

$- \operatorname{sen} \quad g(x)$

$$= -(2x+3)\operatorname{sen}(x^2+3x)$$

$$f(x) = \frac{\operatorname{sen} 2x}{\operatorname{tg} 2x} \qquad f'(x) = \frac{(\operatorname{sen} 2x)' \operatorname{tg} 2x - (\operatorname{sen} 2x)(\operatorname{tg} 2x)'}{(\operatorname{tg} 2x)^2} =$$

$$= \frac{2 \cos 2x \cdot \operatorname{tg} 2x - (\operatorname{sen} 2x)(1+\operatorname{tg}^2 2x)\cdot 2}{(\operatorname{tg} 2x)^2}$$

(TRIGONOMETRÍA)

Sust.

$f'(x_0)$